ME 141
Engineering Mechanics

Lecture 2: Statics of particles

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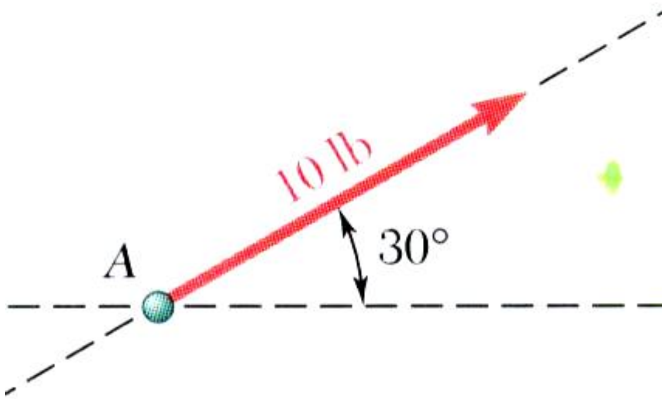
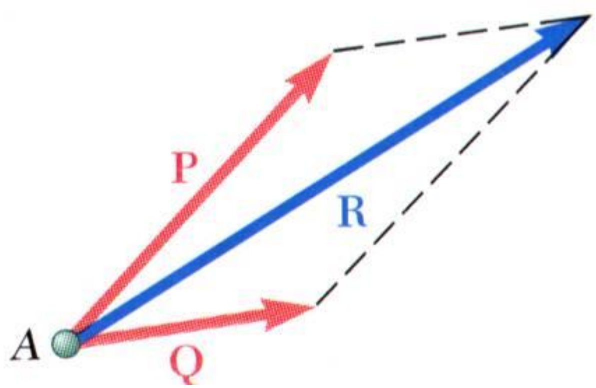
Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Resultant of Two Forces





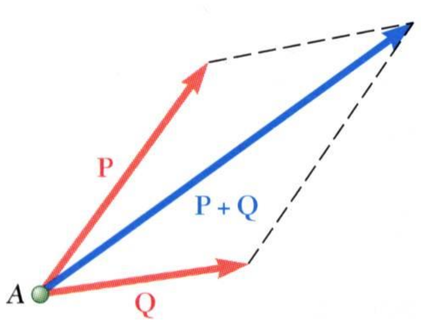
- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.

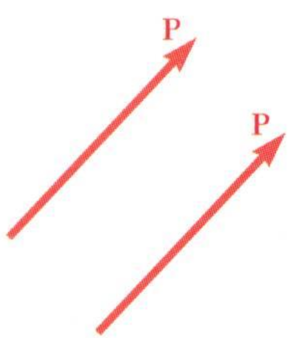
- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.

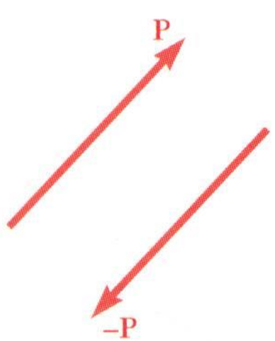
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.

- Force is a *vector* quantity.

Vectors







- *Vector*: parameters possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.

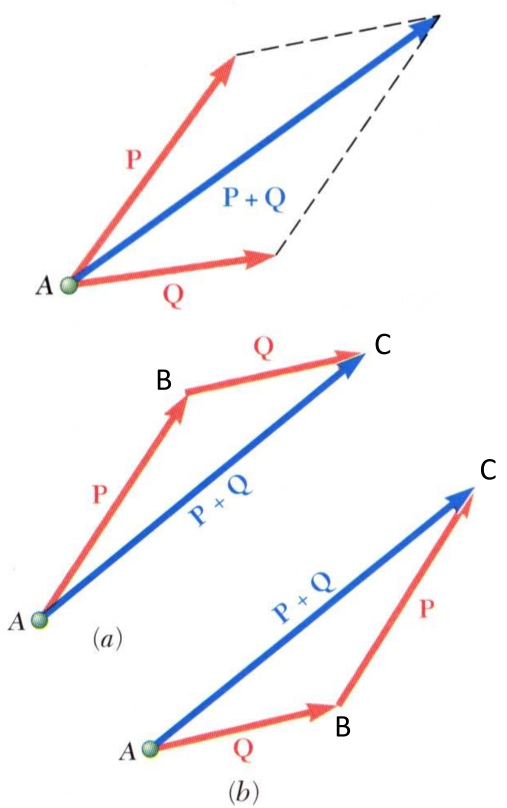
- *Scalar*: parameters possessing magnitude but not direction. Examples: mass, volume, temperature

- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.

- *Equal* vectors have the same magnitude and direction.

- *Negative* vector of a given vector has the same magnitude and the opposite direction.

Addition of Vectors



- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

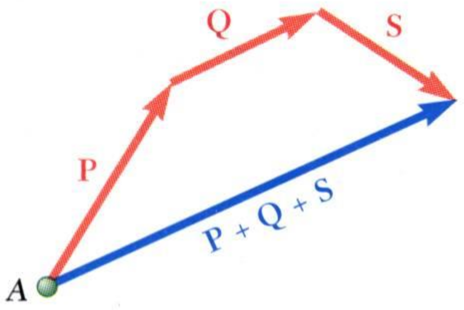
- Law of sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

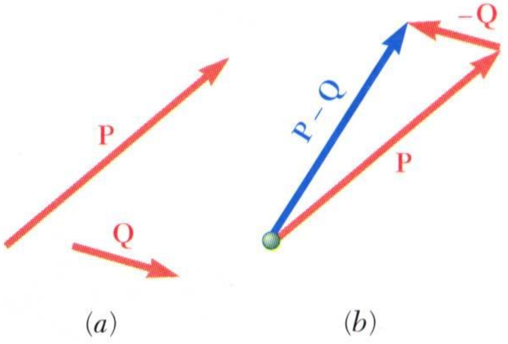
- Vector addition is commutative,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

Addition of Vectors

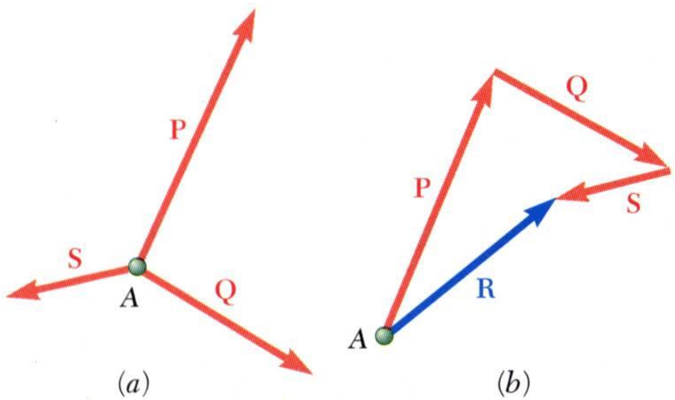


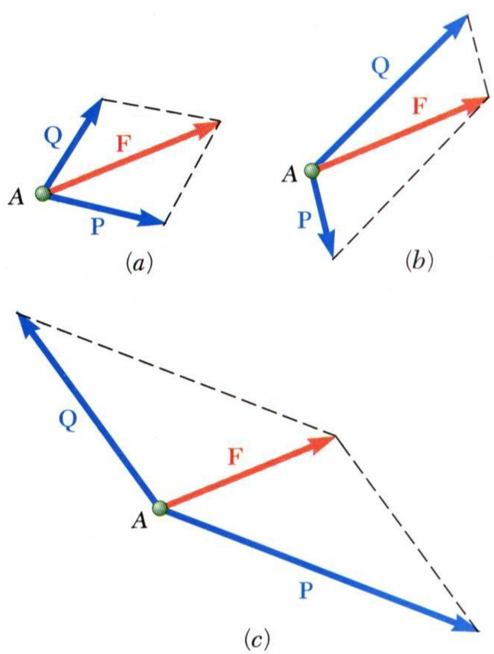
- The polygon rule for the addition of three or more vectors.



- Vector subtraction

Resultant and components of forces



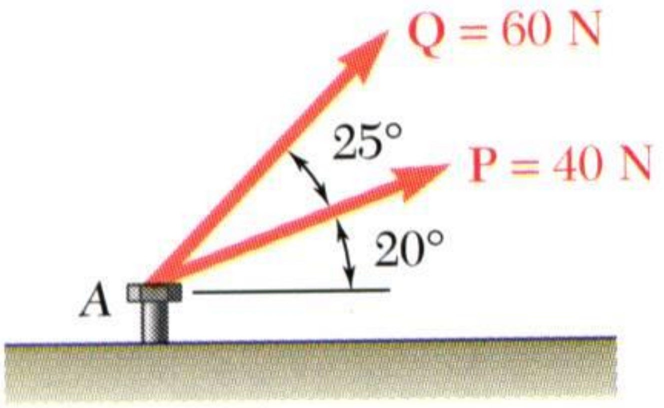


- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

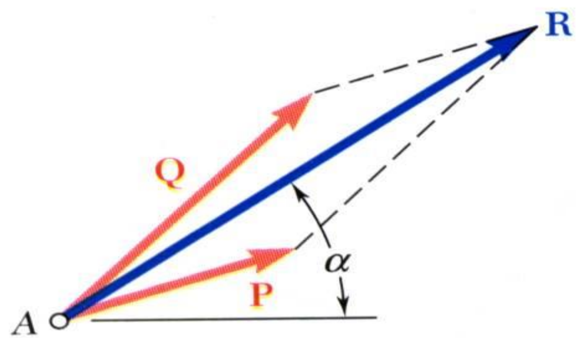
- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Prob# 2.1 (Beer)



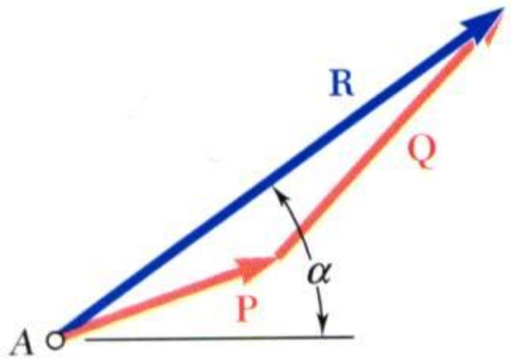
The two forces act on a bolt at *A*. Determine their resultant.

- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,



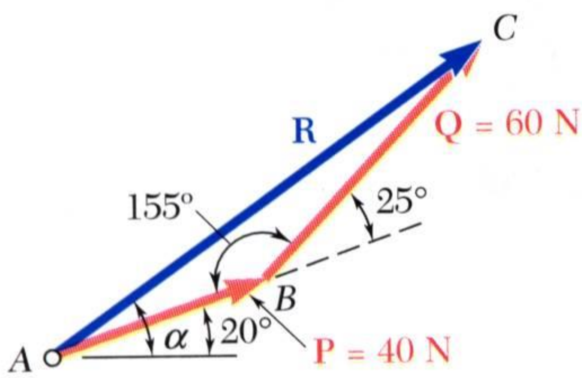
$$\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°$$

Prob# 2.1 (Beer)



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R} = 98\text{ N} \quad \alpha = 35°$$



- Trigonometric solution - Apply the triangle rule. From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$R = 97.73\text{N}$$

From the Law of Sines,

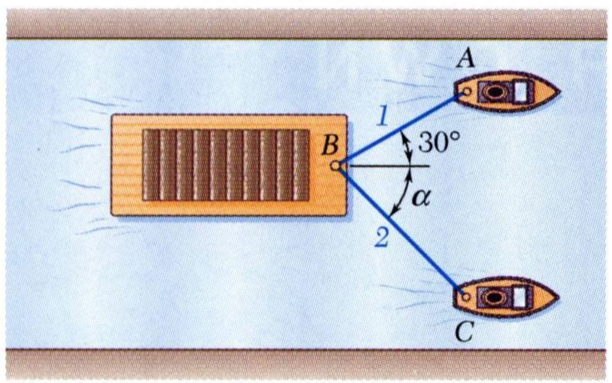
$$\frac{\sin A}{Q} = \frac{\sin B}{R} \quad or, \sin A = \sin B\frac{Q}{R} = \sin 155° \frac{60\text{N}}{97.73\text{N}}$$

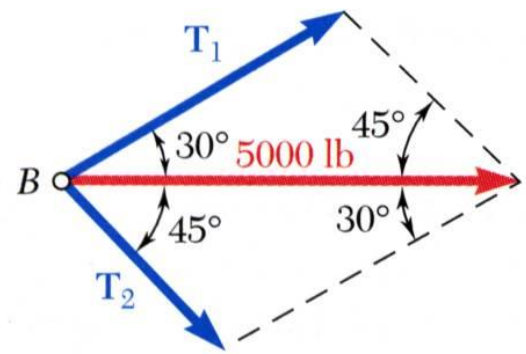
$$A = 15.04°$$

$$\alpha = 20° + A$$

$$\alpha = 35.04°$$

Prob # 2.2 (Beer)





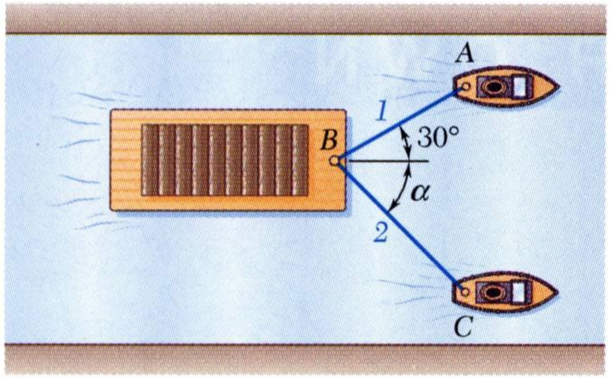
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

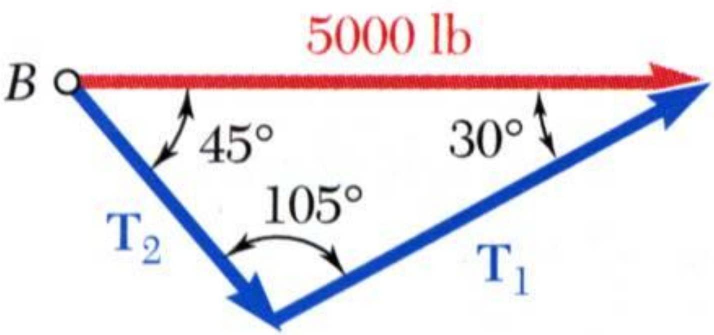
a) the tension in each of the ropes for $\alpha = 45^o$

- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

$$T_1 = 3700\,\text{lbf} \quad T_2 = 2600\,\text{lbf}$$

Prob # 2.2 (Beer)



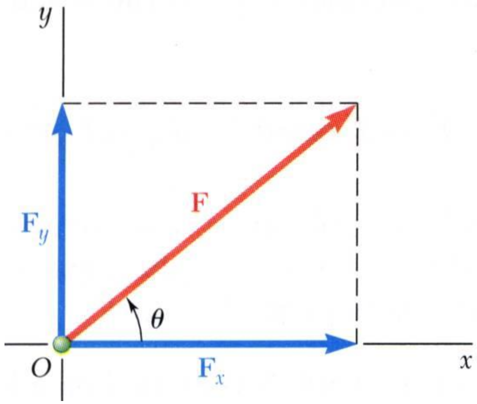


- Trigonometric solution - Triangle Rule with Law of Sines

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

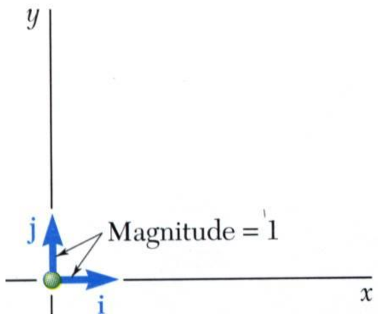
$$\boxed{T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}}$$

Rectangular components of a force

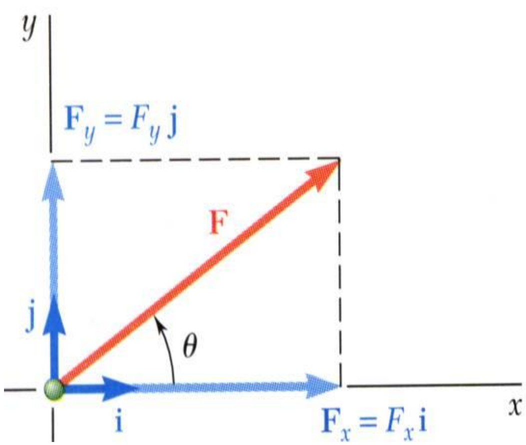


- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$



- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

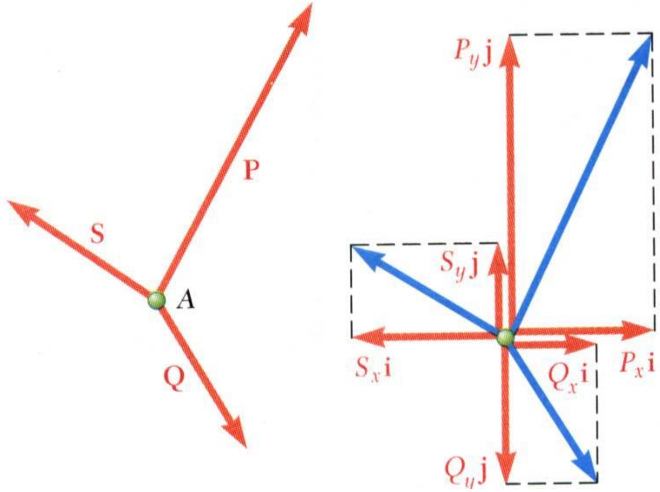


- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of forces by summing components

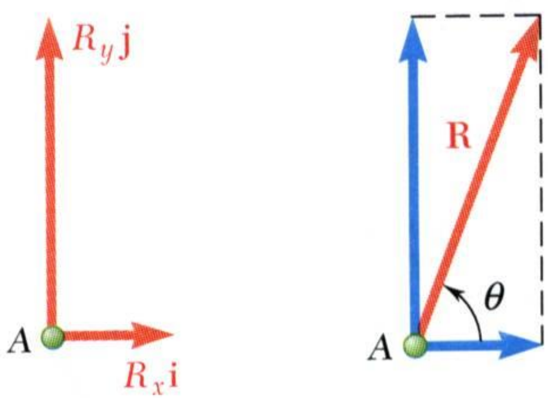


- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

$$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= \left(P_x + Q_x + S_x\right)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$



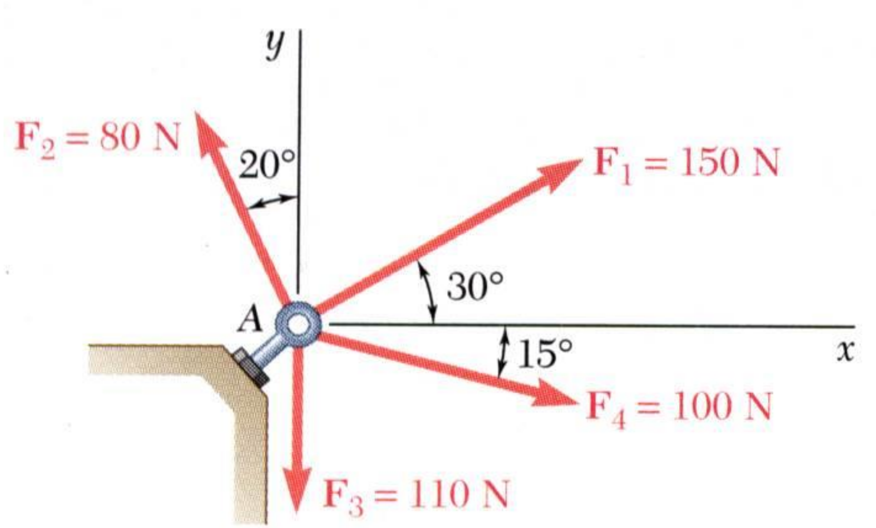
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

$$\begin{aligned} R_x &= P_x + Q_x + S_x \\ &= \sum F_x \end{aligned} \qquad \begin{aligned} R_y &= P_y + Q_y + S_y \\ &= \sum F_y \end{aligned}$$

- To find the resultant magnitude and direction,

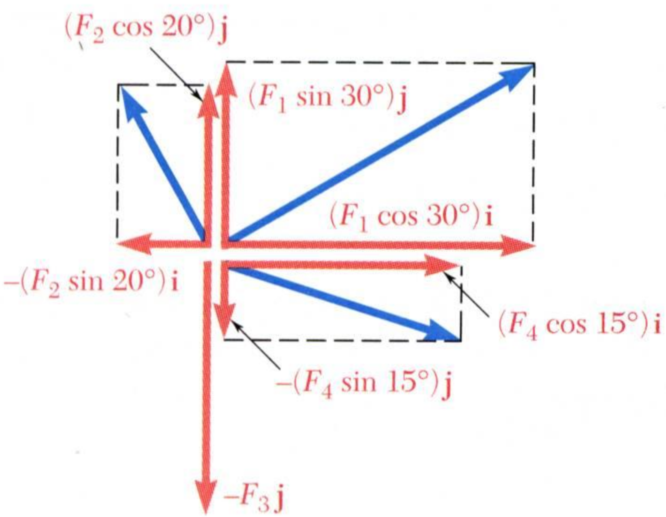
$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

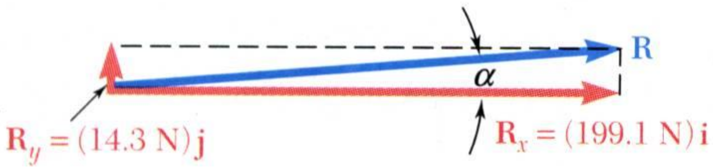
Problem 2.3 (Beer)



Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

Solution





- Resolve each force into rectangular components.

force	*mag*	$x - comp$	$y - comp$
$\vec{F}_1$	150	$+129.9$	$+75.0$
$\vec{F}_2$	80	-27.4	$+75.2$
$\vec{F}_3$	110	0	-110.0
$\vec{F}_4$	100	$+96.6$	-25.9
		$R_x = +199.1$	$R_y = +14.3$

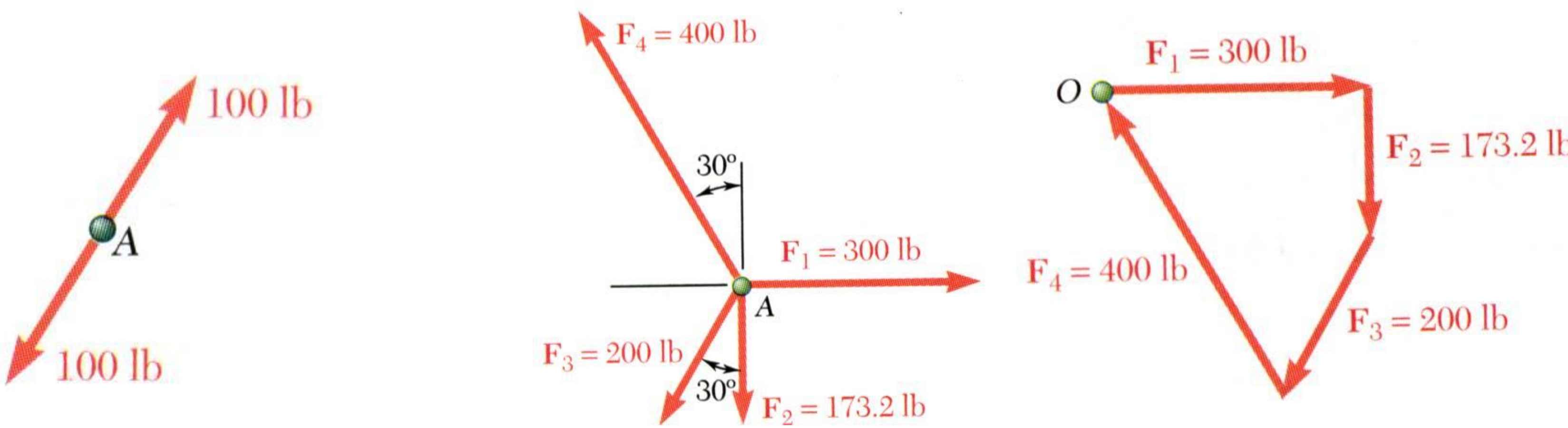
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\text{N}}$$

$$\tan\alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}} \qquad \boxed{\alpha = 4.1°}$$

Equilibrium of a particle

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.

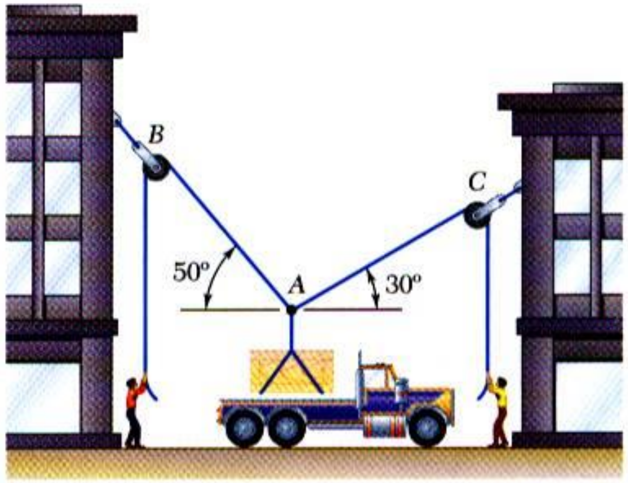


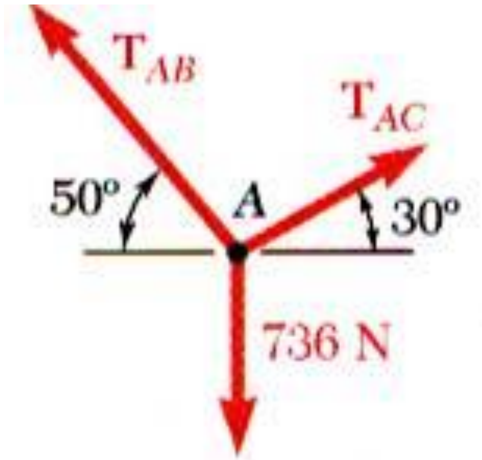
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Free-Body Diagrams

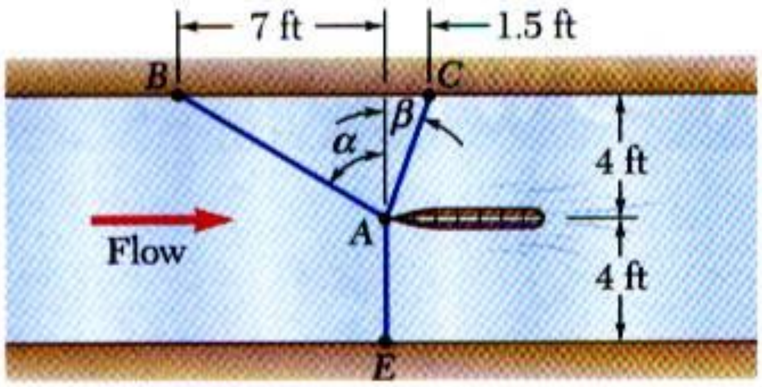




Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

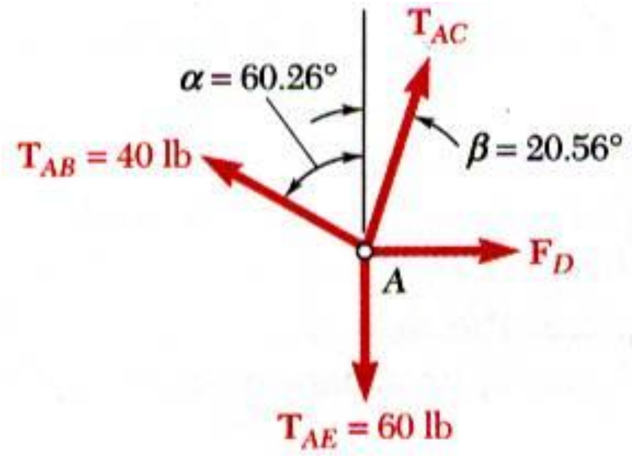
Problem 2.6



It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

Determine the drag force exerted on the hull and the tension in cable *AC*.

Solution



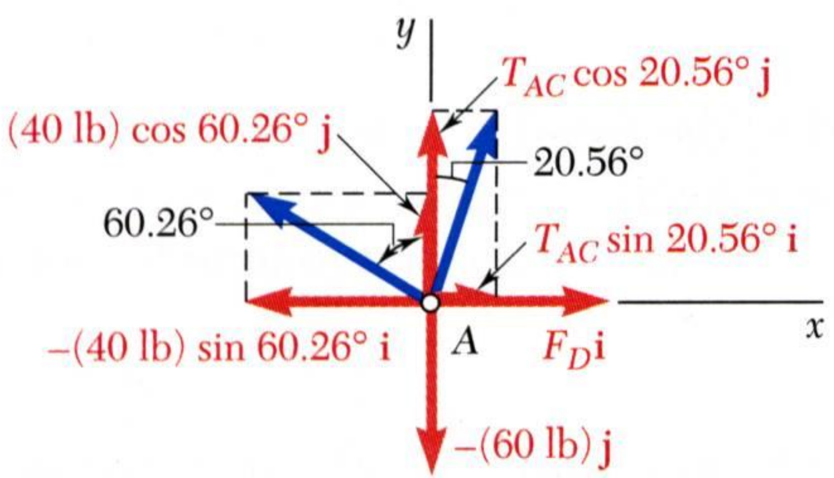
- From the free body diagram of the hull

$$\tan\alpha = \frac{7\text{ ft}}{4\text{ ft}} = 1.75 \qquad \tan\beta = \frac{1.5\text{ ft}}{4\text{ ft}} = 0.375$$

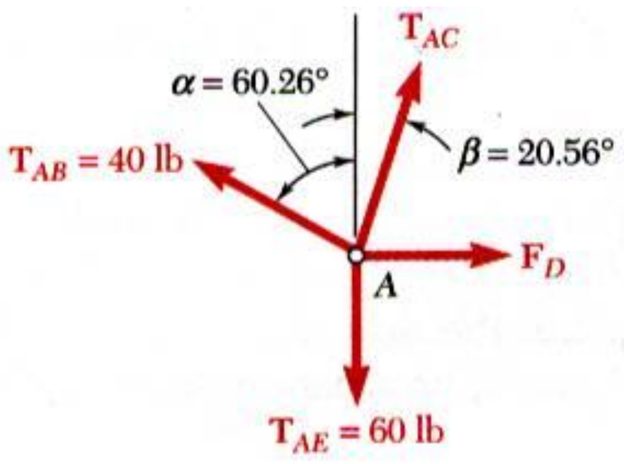
$$\alpha = 60.25° \qquad \beta = 20.56°$$

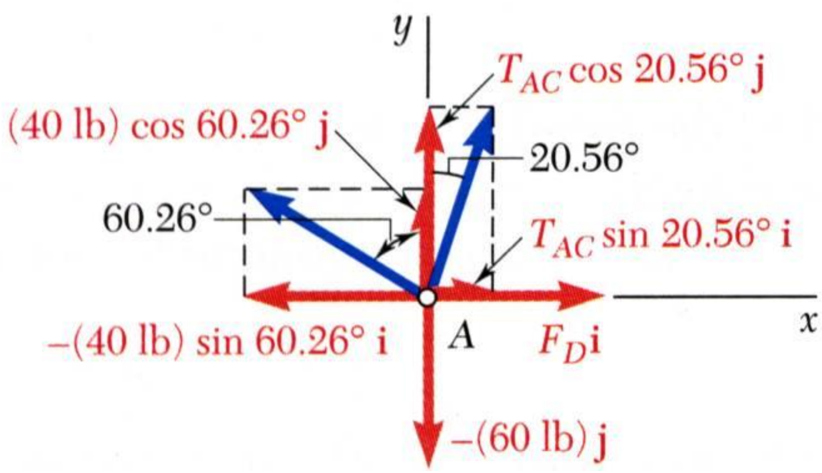
- Now, condition for equilibrium,

$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$



Solution





- Resolving the vector equilibrium equation into two component equations,

$$\bullet \vec{T}_{AB} = -(40\,\text{lb})\sin 60.26°\,\vec{i} + (40\,\text{lb})\cos 60.26°\,\vec{j}$$
$$= -(34.73\,\text{lb})\vec{i} + (19.84\,\text{lb})\vec{j}$$

$$\bullet \vec{T}_{AC} = T_{AC}\sin 20.56°\,\vec{i} + T_{AC}\cos 20.56°\,\vec{j}$$
$$= 0.3512\,T_{AC}\,\vec{i} + 0.9363\,T_{AC}\,\vec{j}$$

$$\bullet \vec{T} = -(60\,\text{lb})\vec{i}$$

$$\bullet \vec{F}_D = F_D\,\vec{i}$$

$$\vec{R} = 0$$
$$= (-34.73 + 0.3512\,T_{AC} + F_D)\,\vec{i}$$
$$+ (19.84 + 0.9363\,T_{AC} - 60)\,\vec{j}$$

Solution

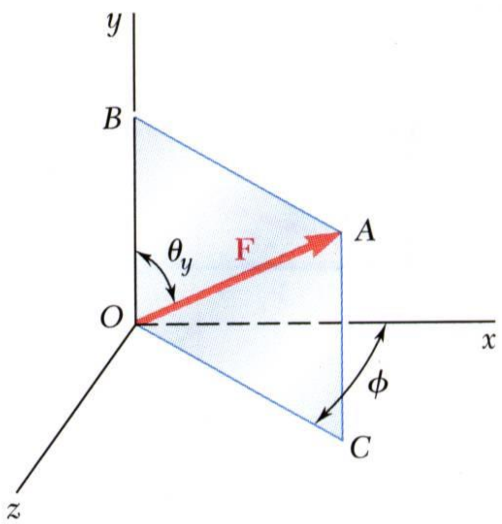
$$\vec{R} = 0$$
$$= \left(-34.73 + 0.3512T_{AC} + F_D\right)\vec{i}$$
$$+ \left(19.84 + 0.9363T_{AC} - 60\right)\vec{j}$$

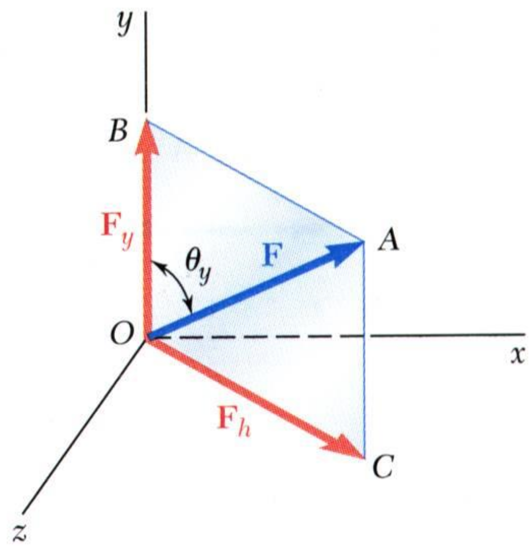
This equation is satisfied only if each component of the resultant is equal to zero

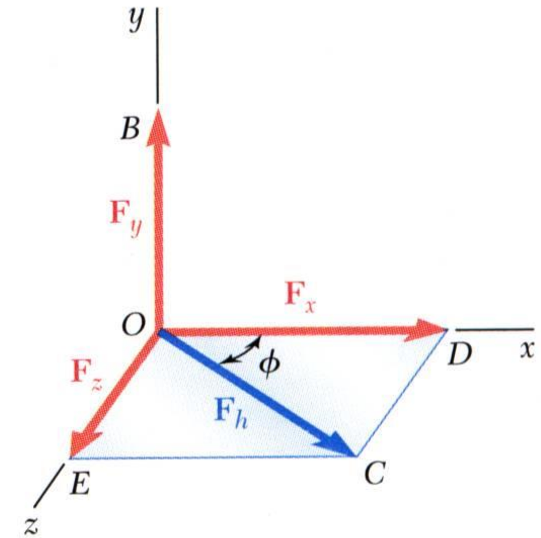
$$\left(\sum F_x = 0\right) \quad 0 = -34.73 + 0.3512T_{AC} + F_D$$
$$\left(\sum F_y = 0\right) \quad 0 = 19.84 + 0.9363T_{AC} - 60$$

$$T_{AC} = +42.9\ \text{lb}$$
$$F_D = +19.66\ \text{lb}$$

Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

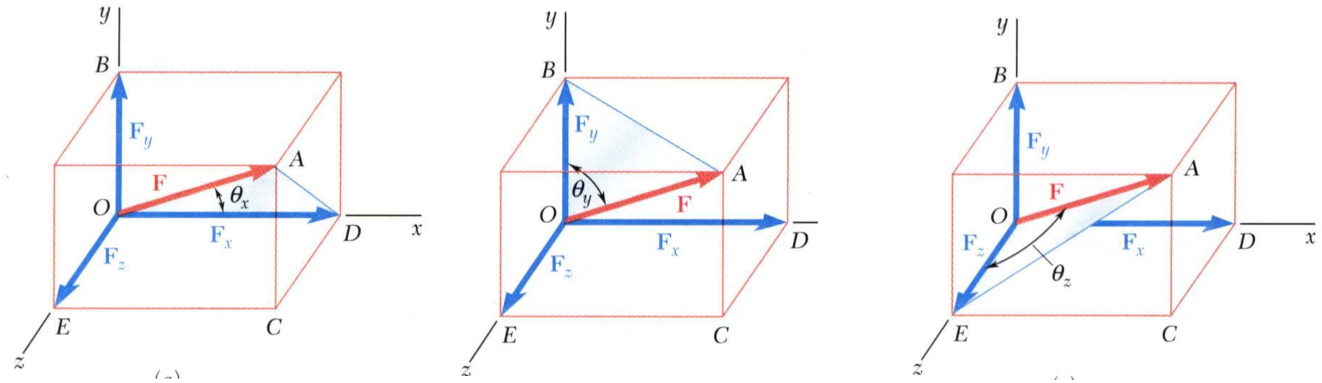
$$F_h = F\sin\theta_y$$

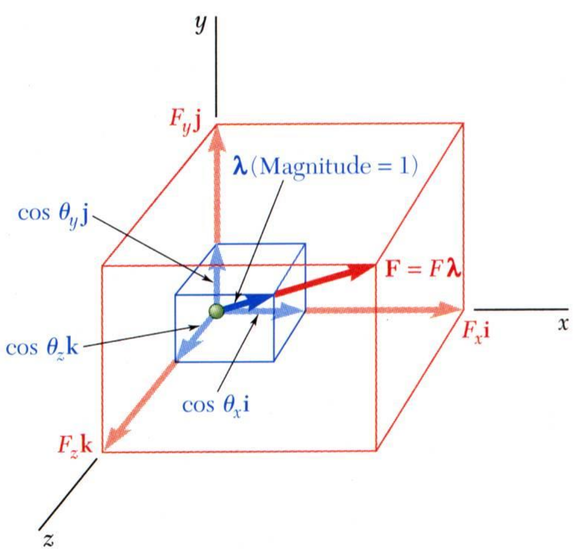
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$$

Rectangular Components in Space





- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

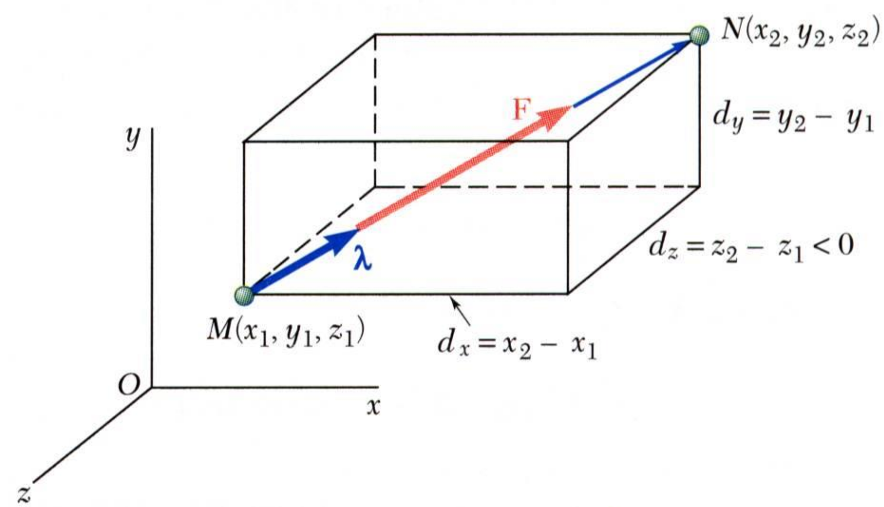
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points,

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = \text{vector joining } M \text{ and } N$$

$$= d_x \vec{i} + d_y \vec{j} + d_z \vec{k}$$

$$d_x = x_2 - x_1, \quad d_y = y_2 - y_1, \quad d_z = z_2 - z_1$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x \vec{i} + d_y \vec{j} + d_z \vec{k}\right)$$

$$\vec{F} = F\vec{\lambda}$$

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Prob# 2.111 (Beer)

- A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 630 lb, determine the vertical force P exerted by the tower on the pin at A.